Math 53, Discussions 116 and 118 **2022-10-14**

Some Midterm 2 Review

Questions

Question 1. Evaluate the limit if it exists:

$$\lim_{(x,y)\to(0,0)} \frac{x^3}{x^4+y^4}.$$

Question 2. Suppose $f(x,y)$ is a differentiable function and $g(u,v) = f(e^u + \cos v, e^u + \sin v)$. Use the values below to calculate $g_u(0,0)$ and $g_v(0,0)$.

$f(0,0) = 3$	$g(0,0) = 6$	$f_x(0,0) = 4$	$f_y(0,0) = 8$
$f(1,2) = 6$	$g(1,2) = 3$	$f_x(1,2) = 2$	$f_y(1,2) = 5$

Question 3. Let $f(x,y) = \sqrt{xy}$.

(a) Compute the gradient of f.
(b) Find the equation of the tangent plane to the graph $z = f(x,y)$ when $(x,y) = (2,8)$.
(c) Find the directional derivative of $f(x,y)$ at $P(2,8)$ in the direction towards the point $Q(5,4)$.

Question 4. Find and classify the critical points of the function

$$f(x,y) = \sin x \sin y$$

Question 5. Find and classify the critical points of the function

$$f(x,y) = x^2 - 4x - y^4.$$

Question 6. Let x, y, z denote the side lengths of a triangle. Heron's formula says that the area of the triangle is

$$A = \sqrt{s(s-x)(s-y)(s-z)}$$

where s is the semiperimeter $s = \frac{1}{2}(x+y+z)$.

Show that if s is fixed, A is maximized when $x = y = z$ (meaning that the triangle is equilateral).

Question 7. Find the absolute maxima and minima of the function $f(x,y) = xy^2$ on the region $x \geq 0, y \geq 0, x^2 + y^2 \leq 3$.